Saving and Growth under Borrowing Constraints
   — Explaining the "High Saving Rate" Puzzle*

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Abstract

Empirical evidence suggests that fast-growing economies tend to have not only high saving rates but also low interest rates. This evidence is difficult to reconcile with standard explanations about the positive linkages between saving and growth. These explanations rely either on high saving to explain high growth or on high growth to explain high saving; but in either case, they must imply and depend on high interest rates to induce high saving rates. Hence, the real puzzle is why households would save excessively to finance firms’ investment when the interest rate on their savings is so low. This paper shows that if households face idiosyncratic wealth-income risk and are borrowing constrained, an otherwise-standard growth model can imply that fast-growing economies have not only high saving rates but also low interest rates. Precautionary saving under borrowing constraints can make an individual’s marginal propensity to consume negatively dependent on her permanent income, so that high income growth can lead to substantially increased saving without high interest rates. The predictions of the model are consistent with the experience of Japan (in the 1960-1970s) and China (in the past 30 years).

Keywords: Saving, Growth, Borrowing Constraints, Permanent Income Hypothesis.

JEL Codes: D91, E21, O4.

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1 Introduction

The permanent income hypothesis (PIH) has had profound influence in shaping modern-day consumption theory and our understanding of the world. However, despite its intuitive appeal, some of its predictions are sharply contradicted by data. One of the predictions of the PIH is that forward-looking consumers should save less in a fast-growing economy because they expect to be richer in the future than they are today. In other words, the saving rate should be lower in a fast-growing economy than in a slow-growing economy.

This prediction based on the PIH has led to the well-known "high saving rate" puzzle for fast growing economies (such as Japan in the 1960-1970s and China in the past 30 years). Much effort has been devoted to resolving this puzzle without reaching a consensus. For example, Horioka (1985, 1990) provides a comprehensive list of various life-cycle factors that may contribute to Japan’s high saving rate, including precautionary saving, housing finance, and income growth. However, careful analysis of the implications of the life-cycle hypothesis leads Hayashi (1986) to reject it as a plausible theory of Japan’s saving behavior. In particular, Hayashi concludes that high income growth cannot explain Japan’s high saving rate because the life-cycle hypothesis depends on heterogeneous cohort effects to generate the positive link between growth and aggregate saving, but such cohort effects are inconsistent with Japanese data.\(^1\)

Yet, empirical evidence suggests that saving and growth are strongly positively correlated across countries, and the positive correlation holds largely because high growth leads to high saving, not the other way around (Carroll, Overland, and Weil, 2000).\(^2\) Nonetheless, most theoretical effort has been devoted to understanding the growth-to-saving causality in a life-cycle framework, because it is thought that this fact is inconsistent with optimal consumption behavior with infinite horizon in a permanent-income framework (see, e.g., Carroll, Overland, and Weil, 2000).

The PIH may mislead us because of the assumption of exogenous, constant rates of returns to financial assets (i.e., the real interest rate). In a production economy with productive assets (such as capital), the real rates of return are determined by the marginal products of such assets. When asset returns are so determined, they will respond to changes in productivity growth, which is the source of permanent income. A permanent increase in the total factor productivity (TFP) raises the rate of return to capital, so investment demand will increase, resulting in a higher equilibrium saving rate through a high real interest rate. Consequently, in contrast to the prediction of the

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1 For alternative explanations of the "high saving rate" puzzle, see Horioka (1990), Gylfason (1993), Carroll, Overland, and Weil (2000), and Wei and Zhang (2009), among others. Also see the recent literature surveys and discussions in Chen, Imrohoroglu, and Imrohoroglu (2006).

PIH, standard general-equilibrium growth theory suggests that household saving should increase rather than decrease in response to a rise in permanent income.

But, fast-growing economies tend to have not just high saving rates, but also low interest rates. For example, in Japan in the 1960-1970s, the household saving rate was as high as 23% and the national saving rate was as high as 25%, while the real 3-month time deposit rate was essentially fixed at below zero.\(^3\) Similarly, in China for the past 30 years, the average personal saving rate has been about 25%, the average national saving rate has been about 40%, yet the real 1-year deposit rate has been less than 5% a year.\(^4\) Hence, besides the puzzle of why high growth can lead to high saving (or vise versa), a bigger puzzle is why high saving is possible when the interest rate is so low, despite high income growth.\(^5\)

This paper argues that borrowing constraints may be a key factor contributing to the observed positive growth-to-saving causality and the high saving rate puzzle.\(^6\) Borrowing constraints can completely alter the relationship between permanent income and consumption by making the marginal propensity to consume a negative function of permanent income, so that high permanent income can lead to an increased propensity to save. As a result, high growth may lead to significantly increased saving without requiring or causing high interest rates.

Consequently, precautionary saving behavior under borrowing constraints can support a large spread between the deposit rate and the rate of return to capital. That is, even if the deposit rate is fixed at zero, the saving rate may still be excessively high and may still respond positively to growth.\(^7\) As a result, high-growth economies may appear to have undiminished high rates of return to capital despite high investment-to-output ratios.

The above arguments are illustrated in this paper using a fairly simple infinite-horizon neoclassical growth model, where long-run income growth is driven by exogenous TFP changes and households are borrowing constrained. To facilitate the analysis, we make some simplifying as-

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\(^3\) An extensive literature argues that the Japanese government repressed interest rates in that period to promote investment and stimulate growth (see, e.g., Horiuchi, 1984; Patrick and Rosovsky, 1976). Dekle (1993) and Homer and Sylla (2005) also note that real interest rates were never particularly high in Japan, but were higher in the low-saving, low-growth 1920's than in the high-saving, high-growth 1960's. Although the rates of return on corporate equities were very high in Japan during that period, households’ share of equities was quite low (see, e.g., Horioka, 1990).

\(^4\) The fact that the real interest rates in China have been kept very low since the economic reform is well known (see, e.g., Zou and Sun, 1996). Since households’ access to financial markets and investment opportunities are typically very limited, plus the extremely high risk of the stockmarket, deposits have been the major means of saving and the most important element explaining China’s high saving rate (see, e.g., Kraay, 2000).

\(^5\) Household saving has been the most important factor driving national saving in both Japan and China in their high-growth periods, during which both economies experienced above 10% average real income growth and low real interest rates. As a reference point, in the standard neoclassical growth model presented in the next section, when the rate of income growth reaches 10% a year, the saving rate will reach about 20%. However, to support this high saving rate, the real interest rate must be about 15%.

\(^6\) Borrowing constraints have been extensively documented in the literature for both developed and developing countries. Households and firms in developing countries are typically far more borrowing constrained and face much higher income uncertainties than those in developed countries. Skinner (1986) presents empirical evidence that income uncertainty is a key factor determining household’s precautionary savings. Also see the arguments in Carroll (1997).

\(^7\) This would not be the case without borrowing constraints. In a standard growth model, the saving rate would decrease with growth if the interest rate were constant, as predicted by the PIH.
sumptions so that the model is analytically tractable. Analytical tractability not only makes the mechanisms transparent but also reduces the computational costs tremendously. First, we assume that the utility function is separable in consumption and leisure. Second, we assume that the marginal utility-cost of leisure is constant, as in Lagos and Wright (2005). Third, we assume that agents are heterogenous due to idiosyncratic wealth-income shocks and that labor supply decisions are made before agents observe their idiosyncratic wealth-income shocks in each period. This last assumption implies that an endogenous labor-supply schedule does not eliminate the buffer-stock role of savings because current wage income cannot be used to buffer the idiosyncratic wealth shocks within the same period. Under these assumptions, the optimal growth model with standard utility and aggregate uncertainty has closed-form solutions for individuals’ decision rules. After aggregation by the law of large numbers, we can investigate the dynamic responses of the economy to both transitory and permanent changes in the growth rate of TFP (the source of permanent income) with or without borrowing constraints.

An important property of the model is that it reduces to a representative-agent, frictionless neoclassical growth model when the distribution of idiosyncratic wealth shocks becomes degenerate. This property makes the model easily comparable to standard models by changing the parameters that control the strength of borrowing constraints.

This paper is closely related to Jappelli and Pagano (1994), Carroll, Overland, and Weil (2000), and Chen, Imrohoroglu, and Imrohoroglu (2006). Jappelli and Pagano (1994) study the relationships among saving, growth, and borrowing constraints in an overlapping-generations model. They show that borrowing constraints can enhance the positive effect of growth on saving. However, as pointed out by Modigliani (1970), Hayashi (1986), and many others, in life-cycle models the positive effect of growth on saving is largely the result of aggregation. To the extent that the economy is growing, workers’ savings will increase relative to retirees’ dissavings, thus, measured aggregate savings will increase. In contrast, this paper studies the issue in an infinite-horizon growth model in which the positive growth-to-saving effect originates from an entirely different mechanism. That borrowing constraints can increase saving is well known, but whether borrowing constraints can also generate and magnify the positive causal effect of growth on saving in an infinite-horizon permanent-income framework is unclear. In this regards, this paper complements the analysis of...

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8Although this assumption implies an infinitely elastic labor supply, it is nonetheless realistic for developing economies such as China—the large rural population in countryside provides an abundant supply of labor. Most importantly, this assumption is necessary for having closed-form solutions. The same assumption is also made by Lagos and Wright (2005) to ensure analytical tractability of their model.

9The solution techniques applied in this paper are similar to those in Wen (2009a, 2009b, 2009c). But these papers assume preference shocks rather than wealth-income shocks. Hence, the distribution of wealth is degenerate in these papers. It is well-known in the literature that, with labor-income shocks, analytical solution is impossible (see, e.g., Imrohoroglu, 1989; Zeldes, 1989; Deaton, 1991; Carroll, 2001; Aiyagari, 1994; Krusell and Smith, 1998). Hence, an independent contribution of this paper is to provide an analytically tractable solution method with wealth-income shocks.
Jappelli and Pagano (1994).\textsuperscript{10}

Alternatively, Carroll, Overland, and Weil (2000) use an infinite horizon endogenous-growth model with habit formation to explain the positive effect of growth on saving. Habit formation can generate and magnify the positive growth-to-saving effect because it makes consumption "sticky". Consequently, an increase in permanent income will raise saving first before having a full impact on consumption. However, in their model high growth implies a high real interest rate, which is one of the key factors leading to a high saving rate.

More recently, Chen, Imrohoroglu, and Imrohoroglu (2006) use a standard neoclassical growth model to offer a quantitative account of the time path of Japan’s saving rate in the postwar period. Their simulations, based on actual time-series data and the assumption of perfect foresight, reveal that stochastic TFP growth is the main force driving Japan’s saving rate. However, these authors also do not address the low interest rate issue.

The rest of the paper is organized as follows. Section 2 presents the model and derives closed-form decision rules for household consumption and saving. Section 3 studies the general-equilibrium effects of growth and borrowing constraints on saving behavior along balanced growth paths. Section 4 studies the short-run dynamics of the aggregate saving rate. Section 5 reconsiders the growth-to-saving effects under a fixed deposit rate. Section 6 shows that the predictions of the model are consistent with the experience of China and Japan during their high-growth periods. Section 7 concludes the paper with remarks for future research. The robustness of the results is analyzed in the Appendix.

2 The Model

There is a unit mass of continuum of households indexed by $i \in [0, 1]$. Taking as given the market real interest rate and real wage, each household chooses sequences of consumption ($C(i)$), savings ($S(i)$), and labor supply ($N(i)$) to maximize expected lifetime utility, $E \sum_{t=0}^{\infty} \beta^t \{ \log C_t(i) - \alpha N_t(i) \}$, subject to the budget constraint $C_t(i) + S_{t+1}(i) \leq [\varepsilon_t(i) + \theta] [(1 + r_t)S_t(i) + W_t N_t(i) + V_t]$, where $r_t$ is the real interest rate, $W_t$ the real wage, and $V_t$ profit income distributed from firms. Note that the level of household wealth-income is multiplied by the term $\theta + \varepsilon_t(i)$, where $\theta \in (0, 1)$ a constant multiplier, and $\varepsilon_t(i)$ an idiosyncratic wealth-income shock. This implies that $\theta$ fraction of the wealth-income is never subject to idiosyncratic shocks. The expected value (mean) of the shock

\textsuperscript{10}The idea that borrowing constraints can lead to excessive saving and can be a possible source of failures of the PIH is hardly new. For example, see Zeldes (1989), Deaton (1991), Carroll (1992, 1997), and Aiyagari (1994, 1995). However, the implications of borrowing constraints for the aggregate relationship between saving and growth have not previously been examined in a rigorous infinite-horizon growth model, to the best of my knowledge. In fact, a recent article by Ludvigson and Michaelides (2001) shows that borrowing constraints are not effective in resolving the "excessively smooth" and "excessively sensitive" consumption puzzles. In addition, none of the above cited works is able to obtain closed-form saving functions under borrowing constraints in an infinite-horizon model as I do in this paper.
is normalized to $E\varepsilon(i) = 1 - \theta$, so that the average value of $\theta + \varepsilon(i)$ equals one and idiosyncratic shocks do not cause distortions to the resource constraint at the aggregate level.\footnote{The multiplicative assumption of individual wealth shocks implies that the degree of inequality in wealth distribution does not shrink but remains stable as the economy grows. This implication is consistent with empirical evidence (see, e.g., Wolff, 1998). The sensitivity of the results to this assumption is analyzed in the Appendix, where I show that the results remain robust when the idiosyncratic shocks are from preferences instead of wealth.}

Households are borrowing constrained; hence we impose $S_{t+1}(i) \geq 0$ in each period. The log form of the utility function ensures the existence of a balanced growth path. This functional form also makes the results comparable to those in Chen, Imrohoroglu, and Imrohoroglu (2006).\footnote{The Appendix shows that the qualitative results do not hinge on the log form of the utility function.} The population is constant over time. Leisure enters the utility linearly as in Lagos and Wright (2005). This assumption serves two purposes. First, with a constant interest rate (i.e., without the general-equilibrium demand-side effect of growth on saving), the optimal level of savings is zero in the absence of idiosyncratic shocks because labor income can always adjust to meet consumption demand when labor is elastically supplied; which helps to isolate the purely precautionary motives of savings under idiosyncratic risk and borrowing constraints. Second and most importantly, this assumption makes the model analytically tractable, as in Lagos and Wright (2005). Without loss of generality, assume $a = 1$.

The information structure and sequence of events are as follows. Within each time period $t$, there are two subperiods. In the first subperiod, households choose labor supply $N_t(i)$ after observing period-$t$ aggregate shocks but without observing the idiosyncratic wealth-income shocks $\varepsilon_t(i)$. In the second subperiod, the idiosyncratic wealth-income shocks are realized and households then choose consumption and savings to maximize expected life-time utilities. This timing structure implies that an endogenous labor-supply schedule does not eliminate the buffer-stock role of savings because current labor income cannot be used to buffer the idiosyncratic wealth-income shocks instantaneously within the same period $t$.

There is a unit mass of continuum of identical firms producing output according to the technology, $Y_t = K_t^{\sigma} (Z_t N_t)^{1-\sigma}$, where $Z_t$ denotes a non-stationary process of labor-augmenting technology, which grows over time according to the process $Z_t = (1 + g_t)Z_{t-1}$. The capital stock ($K$) is accumulated according to $K_{t+1} = (1 - \delta) K_t + I_t$, where $I$ is investment per firm. The stochastic growth rate $g_t$ has mean $\bar{g}$ and follows the law of motion:

$$g_t - \bar{g} = \rho_g (g_{t-1} - \bar{g}) + \mu_t, \quad (1)$$

where the innovation $\mu_t$ is i.i.d with zero mean. When $\bar{g} = 0$ and $\rho_g = 0$, the dynamic effects of $Z_t$ are identical to a random walk TFP shock without drift. Firms behave competitively; hence, the real factor prices are determined by their respective marginal products: $W_t = (1 - \alpha) \frac{Y_t}{N_t}$ and
\[ r_t + \delta = \alpha \frac{Y_t}{K_t}, \] where \( r + \delta \) is the user’s cost of capital with capital depreciation \( \delta \in [0, 1] \). Because of constant returns to scale, the profit income is zero, \( V_t = 0 \).

The model is not stationary in the level but stationary in the growth rate. In the absence of aggregate uncertainty (i.e., \( g_t = \bar{g} \) for all \( t \)), the aggregate economy has a unique constant balanced growth path, along which the real interest rate and aggregate hours worked, \( N_t \), are constant, and the other aggregate variables, such as \( C_t \equiv \int C(i)di, S_{t+1} \equiv \int S(i)di, K_{t+1}, Y_t, \) and \( W_t \) all grow at the same rate \( \bar{g} \). Hence, to solve for the competitive equilibrium, we can transform the model into a stationary one by scaling it down by the growth factor \( (1 + \bar{g})^t \). Using lower-case letters to denote the transformed variables (e.g., \( y_t \equiv \frac{Y_t}{(1+\bar{g})^t} \)), the production function and the real factor prices become

\[
y_t = k_t^\alpha (z_t N_t)^{1-\alpha} \tag{2}
\]

\[
w_t = (1 - \alpha) \frac{y_t}{N_t} \tag{3}
\]

\[
r_t + \delta = \alpha \frac{y_t}{k_t}, \tag{4}
\]

respectively. The capital accumulation equation can be transformed analogously.

After the transformation, household \( i \)'s problem becomes

\[
\max_{\{c,s\}} \mathbb{E}_0 \left\{ \max_{\{N\}} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \log c_t(i) - N_t(i) \right] \right\} \right\}
\]

subject to

\[
c_t(i) + (1 + \bar{g}) s_{t+1}(i) \leq [\varepsilon_t(i) + \theta] \left[ (1 + r_t) s_t(i) + w N_t(i) + v_t \right] \tag{5}
\]

\[
s_{t+1}(i) \geq 0, \tag{6}
\]

where the expectation operator \( \mathbb{E}_t^i \) in the objective function denotes expectations conditional on the information set of time \( t \) excluding \( \varepsilon_t(i) \), and the operator \( \mathbb{E}_t \) denotes expectations based on the full information set in period \( t \) including \( \varepsilon_t(i) \). These notations reflect the information and timing structure of the model. The idiosyncratic \( i.i.d. \) shock has support \( \varepsilon \in [0, \varepsilon_{\max}], \) cumulative distribution function \( F(\varepsilon) \), and mean \( \int \varepsilon dF = 1 - \theta \). The mean requirement implies \( \varepsilon_{\max} > 1 - \theta \).

Idiosyncratic shocks are orthogonal to aggregate shocks.

\[ ^{13} \text{To obtain the equilibrium path of the untransformed variables, we can apply the inverse transformation, such as } Y_t = (1 + \bar{g})^t y_t \text{ and } \frac{Y_{t+1}}{Y_t} = (1 + \bar{g}) \frac{y_t}{y_{t-1}}. \]
Denoting \{\lambda(i), \pi(i)\} as the Lagrangian multipliers for constraints (5) and (6), respectively, the first-order conditions for \{c(i), N(i), s(i)\} are given, respectively, by

\[
\frac{1}{c(i)} = \lambda(i) \tag{7}
\]

\[
1 = w_t E_t^i \{[\varepsilon_t(i) + \theta] \lambda(i)\} \tag{8}
\]

\[
(1 + \bar{g}) \lambda_t(i) = \beta E_t \{(1 + r_{t+1})[\varepsilon_{t+1}(i) + \theta] \lambda_{t+1}(i)\} + \pi_t(i), \tag{9}
\]

where equation (8) reflects the fact that labor supply \(N_t(i)\) is determined before the idiosyncratic wealth shocks (and hence the value of \(\lambda_t(i)\)) are realized. By the law of iterated expectations and the orthogonality assumption of aggregate and idiosyncratic shocks, equation (9) can be written (by using equation 8) as

\[
(1 + \bar{g}) \lambda_t(i) = \beta E_t \frac{1 + r_{t+1}}{w_{t+1}} + \pi_t(i). \tag{10}
\]

The decision rules for an individual’s consumption and savings are characterized by a cutoff strategy, where the cutoff \((\varepsilon_t^*)\) is related to the realization of the idiosyncratic shock. Consider two possible cases:

**Case A.** \(\varepsilon_t(i) \geq \varepsilon_t^*\). In this case the wealth level is high. It is hence optimal to save to prevent possible borrowing constraints in the future when the wealth level may be low. So \(s_{t+1}(i) \geq 0\), \(\pi_t(i) = 0\), and the shadow value of good \(\lambda_t(i) = \beta E_t \frac{1 + r_{t+1}}{(1 + \bar{g})w_{t+1}}\). Equation (7) implies that consumption is given by \(c_t(i) = \left[\beta E_t \frac{1 + r_{t+1}}{(1 + \bar{g})w_{t+1}}\right]^{-1}\). Defining

\[
x_t \equiv (1 + r)s_t(i) + wN_t(i) + v_t \tag{11}
\]
as the wealth (cash in hand) of household \(i\) in the absence of the idiosyncratic shock component, the budget constraint (5) then implies \((1 + \bar{g}) s_{t+1}(i) = [\varepsilon_t(i) + \theta] x_t - \left[\beta E_t \frac{1 + r_{t+1}}{(1 + \bar{g})w_{t+1}}\right]^{-1}\). The requirement \(s_{t+1}(i) \geq 0\) then implies

\[
\varepsilon_t(i) + \theta \geq \frac{1}{x_t} \left[\beta E_t \frac{1 + r_{t+1}}{(1 + \bar{g})w_{t+1}}\right]^{-1} \equiv \varepsilon_t^* + \theta, \tag{12}
\]

which defines the cutoff \(\varepsilon_t^*\).

Notice that the cutoff is independent of \(i\) because wealth \(x_t\) is determined before the realization of \(\varepsilon_t(i)\), and labor supply \(n_t(i)\) can always be adjusted elastically to target any level of wealth under a constant marginal cost of leisure. That is, since all households face the same distribution of
idiosyncratic shocks, the quasi-linear utility function makes it possible in equilibrium each household opts to adjust labor supply to target the same wealth level $x_t$, given the distribution of $\varepsilon(i)$. Hence, equation (11) determines the optimal level of labor supply, given the wealth target $x_t$.

Case B. $\varepsilon_t(i) < \varepsilon_t^*$. In this case the wealth level is low. It is then optimal not to save, so $s_{t+1}(i) = 0$ and $\pi_t(i) > 0$. By the resource constraint (5), we have $c_t(i) = [\varepsilon_t(i) + \theta] x_t$, which by equation (12) implies $c(i) = \frac{\varepsilon_t(i) + \theta}{\varepsilon_t^* + \theta} \left[ \beta E_t \frac{1+r_{t+1}}{(1+\bar{g})w_{t+1}} \right]^{-1}$. Equation (7) then implies that the marginal utility of consumption is given by $\lambda_t(i) = \frac{\varepsilon_t^* + \theta}{\varepsilon_t(i) + \theta} \left[ \beta E_t \frac{1+r_{t+1}}{(1+\bar{g})w_{t+1}} \right]$. Since $\varepsilon(i) < \varepsilon^*$, equation (10) implies $\pi_t(i) = \left[ \beta E_t \frac{1+r_{t+1}}{(1+\bar{g})w_{t+1}} \right] \left[ \frac{\varepsilon^* + \theta}{\varepsilon_t(i) + \theta} - 1 \right] > 0$.

The above analyses imply that the expected shadow value of goods, $E_t \{[\varepsilon(i) + \theta] \lambda(i) \}$, and hence the optimal cutoff value, $\varepsilon^*$, is determined by the following Euler equation for savings based on the first-order condition for labor supply (equation 8):

$$\frac{1}{w_t} = \left[ \beta E_t \frac{1+r_{t+1}}{(1+\bar{g})w_{t+1}} \right] R(\varepsilon_t^*),$$

where

$$R(\varepsilon_t^*) \equiv \theta + \left[ \int_{\varepsilon<\varepsilon^*} \varepsilon^* dF(\varepsilon) + \int_{\varepsilon>\varepsilon^*} \varepsilon dF(\varepsilon) \right].$$

Notice that $R(\varepsilon_t^*) > 1$ because it captures the extra rate of return to savings due to the liquidity value of the buffer stock under borrowing constraints (i.e., a liquidity premium). Hence, savings play the role of liquidity, and the rate of return to liquidity is determined by the real interest rate plus a liquidity premium, $(1+r) R(\theta^*) > 1 + r$.\(^{14}\) It is worth emphasizing again that the cutoff strategy implies that the optimal level of wealth (cash in hand) in period $t$ is determined by a "target" policy given by $x_t = \frac{1}{\varepsilon_t^* + \theta} \left[ \beta E_t \frac{1+r_{t+1}}{(1+\bar{g})w_{t+1}} \right]^{-1}$. Because of a constant marginal cost of leisure, labor supply can adjust elastically to ensure the wealth target $x_t$.

Utilizing the definition of the cutoff in equation (12) and the Euler equation (13), the decision rules of household $i$ are summarized by

$$c_t(i) = \min \left\{ \frac{\varepsilon_t(i) + \theta}{\varepsilon_t^* + \theta}, 1 \right\} \times [\varepsilon_t^* + \theta] x_t$$

and

$$(1+\bar{g}) s_{t+1}(i) = \max \left\{ \frac{\varepsilon_t(i) - \varepsilon_t^*}{\varepsilon_t(i) + \theta}, 0 \right\} \times [\varepsilon_t^* + \theta] x_t$$

\(^{14}\)See Wen (2009a).
\[ x_t = (1 + r_t)s_t(i) + wN_t(i) + v_t = w_t R(\varepsilon_t^*) \frac{1}{\varepsilon_t^* + \theta}. \]  

(17)

Notice that these decision rules are consistent with the budget identity, \( c_t(i) + (1 + g)s_{t+1}(i) = [\varepsilon_t(i) + \theta] x_t \). Hence, the household’s consumption is a concave function of the target wealth, \( [\varepsilon_t^* + \theta] x_t \), with the marginal propensity to consume out of the target wealth given by the function \( \min \left\{ \frac{\varepsilon_t(i) + \theta}{\varepsilon_t^* + \theta}, 1 \right\} \). When the wealth-income shock is low (\( \varepsilon(i) < \varepsilon^* \)), the marginal propensity to consume is less than one; when the wealth-income shock is high (\( \varepsilon(i) \geq \varepsilon^* \)), the marginal propensity to consume equals one. Therefore, saving is a buffer stock: The household saves (\( s_{t+1}(i) > 0 \)) only if the wealth-income shock is high. These properties are consistent with the buffer-stock saving literature (see, e.g., Deaton, 1991; and Carroll, 1992, 1997), except here they are expressed analytically instead of numerically.

Denoting the aggregate variables as \( c \equiv \int c(i)di \), \( s \equiv \int s(i)di \), and \( N \equiv \int N(i)di \), and integrating the household decision rules over \( i \) by the law of large numbers, the policy functions for the aggregate variables are given by

\[ c_t = D(\varepsilon_t^*)x_t \]  

(18)

\[ (1 + \bar{g})s_{t+1} = H(\varepsilon_t^*)x_t \]  

(19)

\[ x_t = (1 + r_t)s_t + wN_t + v_t = w_t R(\varepsilon_t^*) \frac{1}{\varepsilon_t^* + \theta}, \]  

(20)

where

\[ D(\varepsilon^*) \equiv \theta + \int_{\varepsilon(i) \leq \varepsilon^*} \varepsilon(i) dF(\varepsilon) + \int_{\varepsilon(i) > \varepsilon^*} \varepsilon_t^* dF(\varepsilon) \]  

(21)

\[ H(\varepsilon^*) \equiv \int_{\varepsilon(i) > \varepsilon^*} \varepsilon(i) dF(\varepsilon) - \int_{\varepsilon(i) > \varepsilon^*} \varepsilon_t^* dF(\varepsilon), \]  

(22)

and these two functions satisfy \( D(\varepsilon^*) + H(\varepsilon^*) = 1 \) and \( 0 < D(\varepsilon^*) < 1 \).\(^{15}\) Also notice that \( D = 1 + \theta + \varepsilon^* - R \).

Define the disposable income as \( \varphi_t \equiv r_t s_t + w_t N_t + v_t \), which includes labor income, capital gains, and lump-sum profit income. Hence, the aggregate wealth income is related to disposable income by the relation, \( x_t = s_t + \varphi_t \). Using this definition, the consumption function and the saving function together imply the household budget identity,

\[ c_t + (1 + \bar{g}) s_{t+1} - s_t = \varphi_t. \]  

(23)

\(^{15}\)Recall \( E\varepsilon(i) = 1 - \theta \).
Namely, consumption plus net savings (i.e., wealth accumulation) equals disposable income. The aggregate saving rate can thus be defined as the ratio of net savings to disposable income:

\[
\tau_t \equiv \frac{S_{t+1} - S_t}{r_t S_t + W_t N_t} = \frac{(1 + \bar{g}) s_{t+1} - s_t}{\varphi_t}.
\] (24)

In general equilibrium, we have \( s_t = k_t \), \( w_t = (1 - \alpha) \frac{y_t}{N_t} \), and \( r_t + \delta = \alpha \frac{y_t}{k_t} \). Because of constant returns to scale, the profit income \( v_t = 0 \). The household budget identity then becomes

\[
c_t + (1 + g) k_{t+1} - k_t = y_t - \delta k_t,
\] (25)

where \( y_t - \delta k_t = \varphi_t \) is an alternative expression of disposable income. The definition (24) in general equilibrium becomes

\[
\tau_t \equiv \frac{(1 + \bar{g}) k_{t+1} - k_t}{y_t - \delta k_t},
\] (26)

which is identical to the definition of the saving rate adopted by Chen et al. (2006).

The system of equations that determine the general equilibrium of the model consists of equations (13), (18), (19), (25), (2), (3), (4), and a standard transversality condition,

\[
\lim_{T \to \infty} E_0 \beta^T \frac{(1 + \bar{g}) k_{T+1}}{c_T} = 0.
\] (27)

It can be easily confirmed that this dynamic system has a unique saddle path near the steady state; that is, the system has exactly the same number of stable roots as the number of state variables.\(^\text{16}\) Hence, these seven equations plus the transversality condition uniquely solve for the equilibrium path of \( \{\varepsilon_t, c_t, k_{t+1}, N_t, y_t, w_t, r_t\} \), given the distribution of \( \varepsilon_t \), the path of \( \{g_t\} \), and the initial condition \( s_0 \).

### 3 Steady-State Analysis

In the absence of aggregate uncertainty (i.e., \( g_t = \bar{g} \)), the transformed model has a unique steady state in which all aggregate variables are constant. In the steady state, equations (13), (18), (19), and (25) become

\[
1 + \bar{g} = \beta (1 + r) R(\varepsilon^*)
\] (28)

\[
c = D(\varepsilon^*)x
\] (29)

\[
(1 + \bar{g}) k = H(\varepsilon^*)x
\] (30)

\(^{16}\)By equation (13), the cutoff \( \varepsilon_t^* \) is stationary along a balanced growth path as long as the growth rate \( g_t \) is stationary. Hence, the transversality condition is clearly satisfied by the consumption and saving functions.
respectively, where \( x = k + \varphi \). Using equation (30) and realizing that \( H = 1 - D \) gives \( x = \frac{1+\bar{\theta}}{\bar{g} + D} \varphi \).

Substituting this relationship into equations (29) and (31) gives the aggregate consumption and saving as functions of disposable income:

\[
c = \frac{(1 + \bar{g})D}{\bar{g} + D} \varphi \quad \text{(32)}
\]

\[
\bar{g}k = \left[ 1 - \frac{(1 + \bar{g})D}{\bar{g} + D} \right] \varphi. \quad \text{(33)}
\]

Therefore, the marginal propensity to consume out of disposable income is given by \( MPC = \frac{(1 + \bar{g})D}{\bar{g} + D} \), which is less than one because \( D < 1 \), provided that \( \bar{g} > 0 \). The national saving rate is given by

\[
\tau = 1 - \frac{1 + \bar{g} D(\varepsilon^*)}{\bar{g} + D(\varepsilon^*)}. \quad \text{(34)}
\]

The saving rate depends positively on growth \( \bar{g} \). In particular, \( \tau = 0 \) if \( \bar{g} = 0 \), and \( \tau > 0 \) if \( \bar{g} > 0 \).

The following steps solve for the cutoff value as a function of growth:

Equation (28) implies that the output-capital ratio must satisfy

\[
(1 + \bar{g}D) = \beta \left[ 1 - \delta + \alpha \frac{\varepsilon^*}{\bar{k}} \right] R(\varepsilon^*). \quad \text{(32)}
\]

Equations (29) and (30) imply the consumption-capital ratio, \( \frac{c}{k} = \frac{(1 + \bar{g})D}{\bar{g} + D} \). Substituting this consumption-capital ratio into the resource constraint (31) gives another equation for the output-capital ratio:

\[
(\bar{g} + \delta + (1 + \bar{g}) \frac{D}{H}) = \frac{\varepsilon^*}{\bar{k}}. \quad \text{(33)}
\]

Putting these two restrictions for the output-capital ratio together gives the following implicit equation that uniquely solves for the cutoff value \( \varepsilon^* \):

\[
1 + \bar{g} = \beta \left[ 1 - \delta + \alpha \left( \bar{g} + \delta + (1 + \bar{g}) \frac{D(\varepsilon^*)}{H(\varepsilon^*)} \right) \right]. \quad \text{(35)}
\]

Because \( \frac{\partial R(\varepsilon^*)}{\partial \varepsilon^*} > 0 \), the left-hand side (LHS) decreases monotonically with \( \varepsilon^* \). Because \( \varepsilon^* \in [0, \varepsilon_{\text{max}}] \) and \( \varepsilon_{\text{max}} > 1 - \theta \), the LHS has a minimum equal to \( LHS(\varepsilon_{\text{max}}) = \frac{1+\bar{g}}{\theta+\varepsilon_{\text{max}}} < 1 + \bar{g} \) and a maximum equal to \( LHS(0) = (1 + \bar{g}) \). That is, the LHS is a downward-sloping curve with respect to \( \varepsilon^* \). On the other hand, because \( \frac{D(\varepsilon^*)}{H(\varepsilon^*)} \) is monotonically increasing in \( \varepsilon^* \), the right-hand side (RHS) has a maximum equal to infinity at \( \varepsilon^* = \varepsilon_{\text{max}} \) (because \( H(\varepsilon_{\text{max}}) = 0 \)) and a minimum given by \( \beta \left[ 1 - \delta + \alpha \left( \bar{g} + \delta + (1 + \bar{g}) \frac{\theta}{1-\theta} \right) \right] \). That is, the RHS is an upward-sloping curve. Hence, as long as the maximum of the LHS exceeds the minimum of the RHS:

\[
1 + \bar{g} > \beta \left[ 1 - \delta + \alpha \left( \bar{g} + \delta + (1 + \bar{g}) \frac{\theta}{1-\theta} \right) \right], \quad \text{(36)}
\]
a unique interior solution for $\varepsilon^*(\bar{g})$ exists and this value is a function of the growth rate. Condition (36) is clearly satisfied if $\theta = 0$ or if $\theta$ is small enough.

With the cutoff value $\varepsilon^*$ determined, the capital-output ratio can then be derived from equation (28) through the link of interest rate:

$$\frac{k}{y} = \frac{\beta \alpha R(\varepsilon^*)}{1 + \bar{g} - \beta(1 - \delta)R(\varepsilon^*)}. \quad (37)$$

The consumption-to-output ratio is then given by $c/y = 1 - (g + \delta)k/y$. Recall that in a standard, representative-agent neoclassical growth model without borrowing constraints, the steady-state capital-to-output ratio is given by

$$\frac{k}{y} = \frac{\beta \alpha}{1 + \bar{g} - \beta(1 - \delta)}. \quad (38)$$

Hence, at the aggregate level the current model differs from the standard growth model only because the rate of return to cash (liquidity) exceeds one, $R(\varepsilon^*) > 1$, which arises because of a liquidity premium under borrowing constraints. If the variance of the idiosyncratic shocks approaches zero (i.e., the idiosyncratic uncertainty vanishes), then the liquidity premium approaches zero and $R(\varepsilon^*) \to 1$ in the limit. Thus the model reduces to a standard growth model in the limit. This reveals our design of the model: The setup makes it easy to compare this model with standard growth models regarding the influence of borrowing constraints on the growth-saving relationship.

Using equation (37), we can also express the saving rate as

$$\tau = \frac{\bar{g}^k}{1 - \delta k/y} = \frac{\alpha \beta \bar{g} R(\varepsilon^*)}{1 + \bar{g} - \beta(1 - \delta + \alpha \delta)R(\varepsilon^*)}. \quad (39)$$

Clearly, borrowing constraints increase saving because $\frac{d\tau}{dR} > 0$. That is, as the degree of income uncertainty increases, which drives up the liquidity premium, the saving rate will also increase. This explains the numerical findings of Aiyagari (1994). Given the degree of income uncertainty, changes in the growth rate can also affect the liquidity premium, as equation (28) shows. Taking the derivative of the saving rate with respect to $\bar{g}$ gives

$$\frac{d\tau}{d\bar{g}} = \alpha \beta \frac{R[1 - \beta R(1 - \delta + \alpha \delta)] + \bar{g}(1 + \bar{g})\frac{dR}{d\bar{g}}}{[1 + \bar{g} - \beta R(1 - \delta + \alpha \delta)]^2}. \quad (40)$$

Note that, in a standard growth model where $R = 1$ and $\frac{dR}{d\bar{g}} = 0$, we have $\frac{d\tau}{d\bar{g}} > 0$. That is, high growth leads to high saving, as in the data. This explains the findings of Chen, Imrohoroglu,
and Imrohoroglu (2006). However, with borrowing constraints, the sign of expression (40) is difficult to judge, but we can conduct the following informal analysis to obtain insights. Suppose \( \beta R(1 - \delta + \alpha \delta) < 1 \) (which is the case in the standard growth model without borrowing constraints), then the partial derivative of the saving rate with respect to \( \bar{g} \) (taking \( R \) as given) is strictly positive. Also, the partial derivative of \( \tau \) with respect to \( R \) (taking \( g \) as given) is positive. Hence, if in addition we have \( \frac{dR}{dg} > 0 \), then borrowing constraints enhance the positive effect of growth on saving. That is, if the liquidity premium is an increasing function of growth, then borrowing constraints magnify the positive relationship between growth and saving. This turns out to be the case.

Table 1 presents a numerical tabulation of the joint contributions to saving from the two factors: the TFP growth rate (\( \bar{g} \)) and borrowing constraints. To calibrate the model, and for simplicity, we assume \( \varepsilon_t(i) \) follows the power distribution function \( F(\varepsilon) = \left( \frac{\varepsilon(i)}{\varepsilon_{\text{max}}} \right)^\sigma \), with support \( \varepsilon(i) \in [0, \varepsilon_{\text{max}}] \) and the shape parameter \( \sigma \in (0, \infty) \). We set the upper-bound parameter \( \theta_{\text{max}} = \frac{1 + \sigma}{\sigma} (1 - \theta) \) so that the average value of \( \varepsilon(i) \) is \( E\varepsilon = 1 - \theta \). With this specification, we have

\[
R(\varepsilon^*) = 1 + \frac{1}{1 + \varepsilon_{\text{max}}^\sigma \varepsilon^* \varepsilon^{1+\sigma}} \tag{41}
\]

\[
D(\varepsilon^*) = \theta + \varepsilon^* \left[ 1 - \frac{1}{1 + \varepsilon_{\text{max}}^\sigma \varepsilon^* \varepsilon^{\sigma}} \right] \tag{42}
\]

\[
H(\varepsilon^*) = 1 - \theta - \varepsilon^* \left[ 1 - \frac{1}{1 + \varepsilon_{\text{max}}^\sigma \varepsilon^* \varepsilon^{\sigma}} \right]. \tag{43}
\]

We calibrate the model’s structural parameters as follows: The time period is a year, the time discounting factor \( \beta = 0.96 \), the output elasticity of capital \( \alpha = 0.4 \), and the rate of capital depreciation \( \delta = 0.1 \). The value of \( \sigma \) in the power distribution function and the constant \( \theta \) are chosen so that the steady-state distribution of household wealth in the model, \([\varepsilon(i) + \theta] x\), roughly matches that of the Asian developing countries in terms of the Gini coefficient. This gives \( \theta = 0.1 \) and \( \sigma \simeq 0.15 \).\(^{17}\) The calibrated parameter values are summarized in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>0.96</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.4</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.1</td>
</tr>
<tr>
<td>( \theta )</td>
<td>0.1</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>0.15</td>
</tr>
</tbody>
</table>

\(^{17}\)The Gini coefficient implied by wealth distribution is about 0.55 for China, 0.71 for Thailand, 0.76 for Indonesia, 0.65 for Taiwan, and 0.58 for South Korea. The average is about 0.65 (see, e.g., Davies, Sandstrom, Shorrocks, and Wolff, 2006). \( \sigma \) is chosen to match this average number. For comparison, the Gini coefficient in the United States is 0.8. Notice that the larger the Gini coefficient, the stronger the growth-to-saving effect in our model.
The predicted saving rates are reported in Table 2, where Model A represents the counterpart representative-agent neoclassical growth model without borrowing constraints (called a "standard growth model" in this paper), and Model B represents the heterogeneous-agent model with borrowing constraints. The table shows that high growth leads unambiguously to high saving in both models, regardless of borrowing constraints. For example, in the representative-agent growth model without borrowing constraints (Model A), when the growth rate is at 1% per year, the saving rate is less than 4%; however, when the growth rate increases to 10% per year, the saving rate rises to nearly 20%. Thus, theory predicts that high growth leads to high saving, which is consistent with the data, but in sharp contrast to PIH.

The fundamental reason that high growth leads to high saving in the standard growth model is that growth enhances the productivity of capital, which raises the demand for investment, which in turn raises the interest rate and consequently leads to increased saving in equilibrium. Conversely, the conventional PIH is presented in a partial-equilibrium framework with a constant interest rate, so high growth is not accompanied by high asset returns; thus, consumers have no incentives to increase saving, but opt to raise their marginal propensity to consume when permanent income rises.

Table 2. Saving Rate ($\tau$) as a Function of Mean Growth ($\bar{g}$)

<table>
<thead>
<tr>
<th>$\bar{g}$</th>
<th>1%</th>
<th>2%</th>
<th>3%</th>
<th>4%</th>
<th>5%</th>
<th>6%</th>
<th>7%</th>
<th>8%</th>
<th>9%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model A</td>
<td>3.6%</td>
<td>6.5%</td>
<td>9.0%</td>
<td>11.2%</td>
<td>13.0%</td>
<td>14.6%</td>
<td>16.0%</td>
<td>17.3%</td>
<td>18.4%</td>
<td>19.4%</td>
</tr>
<tr>
<td>Model B</td>
<td>5.4%</td>
<td>10%</td>
<td>13.9%</td>
<td>17.2%</td>
<td>20.1%</td>
<td>22.6%</td>
<td>24.9%</td>
<td>26.9%</td>
<td>28.7%</td>
<td>30.3%</td>
</tr>
</tbody>
</table>


Borrowing constraints can significantly amplify the growth effect on saving. Table 2 shows that with borrowing constraints, the saving rate not only is much higher than that in the standard model at each level of growth rate, but the gap also increases with the growth rate. For example, when the growth rate is at 1% per year, the saving rate is 5.4% in Model B, which is 1.8 percentage points higher than that in Model A. When the growth rate is at 10% per year, the saving rate is 30.3% in Model B, which is about 11 percentage points higher than that in Model A. This implies that borrowing constraints can greatly magnify the positive effect of growth on saving.\(^{18}\)

The results of Table 2 are graphed in the left-hand window in Figure 1, where the line with circles represents the standard growth model (Model A), and the line with triangles the model with borrowing constraints (Model B). It shows that (i) high growth leads to increased saving regardless

\(^{18}\)Borrowing constraints \textit{per se} will induce a higher saving rate because of the buffer-stock role of savings, other things equal. However, if there were no amplification effects, borrowing constraints would only generate a constantly higher saving rate than the standard growth model when the growth rate rises, instead of an increasingly higher rate, as shown in Table 1.
of borrowing constraints and (ii) borrowing constraints enhance saving and that this enhancing effect is larger the higher the growth rate.

![Figure 1. The Growth Effects on Saving and Interest](image)

Borrowing constraints not only significantly magnify the growth effect on saving, but also mitigate the growth effect on interest rates. The lower panel in Table 3 (Model B) shows that with borrowing constraints, the real interest rate not only is significantly lower than that in the standard growth model at every level of growth, but also increases less rapidly with growth. For example, when the growth rate is 1% to 3%, the implied interest rate is about 5% to 7% without borrowing constraints (Model A), but only about 1% to 3% with borrowing constraints. Also, when the growth rate rises to 8% to 10%, the implied interest rate jumps up to 13% to 15% without borrowing constraints (Model A), but increases only to about 6% to 7% with borrowing constraints (Model B).
Table 3. Equilibrium Interest Rate ($r$) as a Function of Mean Growth ($g$)

<table>
<thead>
<tr>
<th>$g$</th>
<th>1%</th>
<th>2%</th>
<th>3%</th>
<th>4%</th>
<th>5%</th>
<th>6%</th>
<th>7%</th>
<th>8%</th>
<th>9%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model A</td>
<td>5.2%</td>
<td>6.3%</td>
<td>7.3%</td>
<td>8.3%</td>
<td>9.4%</td>
<td>10.4%</td>
<td>11.5%</td>
<td>12.5%</td>
<td>13.5%</td>
<td>14.6%</td>
</tr>
<tr>
<td>Model B</td>
<td>1.3%</td>
<td>2.0%</td>
<td>2.7%</td>
<td>3.3%</td>
<td>4.0%</td>
<td>4.6%</td>
<td>5.3%</td>
<td>5.9%</td>
<td>6.6%</td>
<td>7.2%</td>
</tr>
</tbody>
</table>

Model A: without borrowing constraints. Model B: with borrowing constraints.

The results of Table 3 are graphed in the right-hand window in Figure 1, where triangles represent the model with borrowing constraints and circles the model without borrowing constraints. Clearly, not only does the line with borrowing constraints lie significantly below the line without borrowing constraints at all levels of growth rates, its slope is also less steep.

4 Dynamic Analysis

This section examines the dynamics of the saving rate in two scenarios. In the first scenario, we study behaviors of the saving rate and its relation to growth when an economy starts out "poor", in the sense of having a capital stock below the steady state. We show that the model with borrowing constraints will have not only a higher growth rate but also a higher saving rate along the path converging to the steady state than the model without borrowing constraints, although both economies share the same steady state and rate of TFP growth. Since different degrees of borrowing constraints lead to different steady states, to ensure that they share the same steady state, we assume that in the borrowing constrained economy the constraints are gradually reduced (by decreasing the variance of the idiosyncratic shocks, $\sigma$) along the transitional path so that they no longer bind in the long run.\(^{19}\) The other parameters are calibrated to the same values as shown in Table 1, and the steady-state rate of TFP growth is set to $\bar{g} = 0.04$.

Figure 2 depicts the growth rate of investment and the saving rate as a function of the capital stock for the two economies. The dots represent equally spaced points in time as the system evolves toward the shared steady state. The triangle symbols represent the model with borrowing constraints, and circles the counterpart model without borrowing constraints. Both models start out with the same level of capital stock, but the model with borrowing constraints starts out with $\sigma = 0.15$ in the first period and this value increases over time (in every subsequent period) until it becomes large enough so that the two models converge to the same steady state in the long run.

Figure 2 shows that an economy that starts out poor will initially have both a higher-than-steady-state saving rate and a higher-than-steady-state growth rate, regardless of borrowing constraints. However, the borrowing constrained economy will exhibit a much higher saving rate and a moderately higher growth rate than the counterpart economy at any point in time.\(^{20}\) For example, the saving rate is about 7 percentage points higher with borrowing constraints than without in the

\(^{19}\) $\sigma = \infty$ in the model without borrowing constraints.

\(^{20}\) Since time is discrete, we assume that in the initial period the growth rate equals the steady-state rate.
initial period. The intuition for this pattern is that borrowing constraints induce excessive saving, which results in more rapid capital accumulation and higher growth along the transitional path, even though the TFP growth rate is the same (4% a year) across the two economies.

Figure 2. Transitional Dynamics

In the second scenario, we study the impulse responses of the saving rate to transitory TFP-growth shocks in the transformed economies. We use the growth process specified in equation (1) to drive the model by setting the mean annual growth rate to $\bar{g} = 0.04$ with persistence $\rho_g = 0.2$, consistent with postwar U.S. data.\footnote{The qualitative results (i.e., the directions of changes) are robust to these parameter values.} Figure 3 shows the impulse responses (percentage deviations) of the saving rate ($\tau_i$) to a one-standard-deviation transitory increase in the growth rate, $g_t$. Since the transformed model is solved by the method of log-linearization around steady state, all changes in Figure 3 represent percentage deviations relative to the steady state.

The top-left window shows the impulse responses of the saving rate in the standard growth model, and the bottom-left window shows them in the borrowing constrained model. In either
case, the saving rate rises after a positive shock to TFP growth. This happens because the higher rate of returns to capital induces investment demand and stimulates saving. Therefore, in sharp contrast to the prediction of the PIH, households increase rather than decrease their saving when permanent income rises, even though the higher income growth is purely transitory.

The above results are not sensitive to the definition of the saving rate adopted in this paper (equation 26). For example, suppose the saving rate is defined as the ratio of gross investment to output ($\tau_t = \frac{I_t}{Y_t} = \frac{(1+g)k_{t+1} - (1-\delta)k_t}{y_t}$), it still responds positively to a growth shock, regardless of borrowing constraints (see the top-right window and bottom-right window in Figure 3). This suggests that consumption growth responds less than one-for-one to income growth, whereas investment growth responds more than one-for-one to income growth. Hence, once the interest rate is endogenous, there does not exist the so called "excess smoothness puzzle" of consumption relative to income.\(^{22}\)

![Figure 3. Impulse Responses of Saving to 1% Growth Shock](image)

However, here arises another puzzle: In the general-equilibrium models presented above, the real interest rate is also the rate of return to capital. The empirical evidence suggests that for fast-

\(^{22}\)For more analysis on the "excess smoothness puzzle", see Wen (2009) and the literature therein.
growing emerging economies the rate of return to capital may be extremely high while the interest rates facing households may be extremely low. For example, the average 1-year nominal deposit rate in China was about 5% per year from 1991 to 2007, yet the average rate of return to capital was more than 20% per year in that period (Bai, Hsieh, and Qian, 2006), whereas the household saving rate in that period was about 25%, the national saving rate was about 40%, and the average growth rate of real GDP per capita was about 10% per year. The situation is similar in Japan. During the high-growth and high-saving period of Japan in the 1960-1970s, the average nominal 3-month deposit rate was about 4% and the average after-tax real rate of return to capital was above 16%, whereas the average household saving rate was about 17%, the national saving rate was about 20%, and the average growth rate of real income was above 10%.

In both economies during their respective high-growth period, bank deposits have been the major means of saving for households, as well as the most important source of funds for firms' investment. For example, in China, bank deposits accounted for 72% of total household financial assets in year 2004 and 2005. In contrast, the total share of bonds and stocks accounted for less than 10% in that period. On the other hand, the share of bank loans in total corporate debt was about 64% in 2004 and 2005, while the total share of corporate bonds and stocks was only around 15% in that period.

Therefore, regardless of how interest rates are kept low in fast-growing economies, the puzzle is not just why saving is high under high growth, but also why it is high under low interest rates. Namely, why would households save excessively to finance firms' investment when returns to their savings are so low and do not reflect TFP growth? We address this puzzle in the next section.

5 The Cases of Low Deposit Rates

In developing economies, because of incomplete markets and various forms of financial repression (including distorted government banking regulations and monetary policies), there may exist large spreads between deposit rates that households get for their savings and the true rates of returns to capital that firms get for their investment. If such spreads exist, how do they affect the relationship between saving and growth? This section analyzes this issue by conducting a counter-factual experiment.

Suppose that the real interest rate faced by households is not the same as the marginal product of capital. In particular, suppose households have no access to investment opportunities except earning a low, fixed real interest rate ($\bar{r}$) on their deposits at financial intermediaries. On the other
hand, firms must pay the market real interest rate \((r_t)\) to obtain loans, and banks earn monopolistic
profits from the spread in rates of returns, \((r_t - \bar{r})s_t \geq 0\). For simplicity, assume that the profits
are nonetheless redistributed as a lump sum to households: \(v_t = (r_t - \bar{r})s_t\).

This model with the spread in the rates of returns can be solved in exactly the same way as
discussed in section 2. In equilibrium, we still have the capital market clearing condition, \(s_t = k_t\);
namely, the supply of capital determines its demand.\(^{26}\) In the steady state, we have the following
analogous relationships and decision rules:

\[
1 + \bar{g} = \beta(1 + \bar{r})R(\varepsilon^*) \tag{44}
\]
\[
c = D(\varepsilon^*)[(1 - \delta)k + y] \tag{45}
\]
\[
(1 + \bar{g})k = H(\varepsilon^*)[(1 - \delta)k + y] \tag{46}
\]
\[
c + \bar{g}k = y - \delta k \tag{47}
\]
\[
r = \alpha\frac{y}{k} - \delta. \tag{48}
\]

The above equations imply that the saving rate is still determined by the formula in equation (34):
\[
\tau = 1 - \frac{(1 + \bar{g})D}{\bar{g} + D}. \tag{34}
\]
However, the implied value of the saving rate given by this equation now differs
from that given by equation (39) because the deposit rate \((\bar{r})\) in equation (44) is no longer equal
to the marginal product of capital. Hence, the value of cutoff \((\varepsilon^*)\) determined by equation (44)
differs from that in the previous model. In particular, with a fixed deposit rate below the market
rate, the optimal cutoff \(\varepsilon^*\) is higher and increases faster as \(\bar{g}\) rises. That is, for any given level of
the growth rate, the portion of the population with positive saving is smaller in the current model
because of utility maximization from households.

Notice that if we impose the rate-of-return spread on the standard growth model without
borrowing constraints, then the saving rate would be constant and would not respond to growth
because the interest rate is completely de-linked from the marginal product of capital. However,
this is not the case under borrowing constraints.

In Figure 4, the line with squares in the left-hand window shows the relationship between saving
and growth when the real deposit rate equals 1\% per year.\(^{27}\) For comparison, in Figure 4 we also
include the same curves shown in Figure 1. The left-hand window in Figure 4 shows that, even
with such a low and fixed deposit rate, households have significantly higher saving rates than those
in the standard growth model (the line with circles) at various levels of the growth rate, although

\(^{26}\) Because of the below-equilibrium deposit rate, the demand of capital is rationed along the saving curve. This
implies that the marginal product capital exceeds the real deposit rate. However, the profits from the spread are
eraned by banks because firms still pay interest rates equal to the marginal products of capital. Alternatively, we
can also allow firms to earn the profits.

\(^{27}\) The results do not change qualitatively even if the real deposit rate is zero or negative.
they are lower than those in the counterpart model without the distorting spread (the line with triangles). For example, when the growth rate is about 3% a year, the saving rate in the standard model without borrowing constraints is about 9%; however, this saving rate is about 13% in the current model despite an essentially zero deposit rate. Therefore, income uncertainty and borrowing constraints are able to generate an excessive amount of savings even under low interest rates.

More importantly, thanks again to borrowing constraints, as the growth rate increases, the saving rate also rises accordingly, despite the fixed low deposit rate. That is, even though TFP growth does not transmit to the rate of returns to household savings, the saving rate still increases with economic growth. This is because TFP growth impact on real wage, and borrowing constraints make the marginal propensity to save positively dependent on permanent income. Hence, even if the real interest rate is fixed at extremely low levels, households still prefer raising their saving-to-income ratio when permanent income increases. Similar results hold even if the real deposit rate is negative.

Figure 4. Effects of Fixed Deposit Rate
On the other hand, the right-hand window in Figure 4 shows that the real rate of return to capital in the current model with a fixed deposit rate (the line with square symbols) is higher than that in the counterpart model without the distorting spread (the line with triangles), and it rises faster when the growth rate increases. This explains why fast-growing economies may simultaneously exhibit low deposit rates and high (and apparently undiminished) rates of returns to capital.

The property that the marginal propensity to save depends positively on the level of permanent income is the sole consequence of borrowing constraints. That is, borrowing constraints can completely alter the PIH and generate exactly the opposite prediction of the PIH. This powerful effect of borrowing constraints can be even seen in a partial-equilibrium framework where there is no capital and both the real interest rate and the real wage are exogenous.

For example, consider the special case where there is no capital and \( \alpha = 0 \). The production function then becomes \( Y_t = Z_t N_t \). The real wage then becomes completely exogenous, \( W_t = Z_t \), which grows over time at the rate \( g_t \). Let the real interest rate on saving be given by the constant \( r \). This partial-equilibrium model has steady-state solutions analogous to equations (44) to (47):

\[
1 + \bar{g} = \beta (1 + \bar{r}) R(\bar{\varepsilon}) \\
c = D(\bar{\varepsilon}) x \\
(1 + \bar{g}) s = H(\bar{\varepsilon}) x \\
c + \bar{g} s = \varphi, \tag{52}
\]

where \( x = s + \varphi \) and \( \varphi = \bar{r} s + wN + v \). Hence, the saving rate is still given by the same formula, \( \tau = 1 - \frac{(1+\bar{g})D}{\bar{g}+D} \), and the relationship between saving and growth is identical to that implied by a fixed deposit rate graphed in the left-hand window in Figure 4 (i.e., the line with squares).

This partial-equilibrium analysis reveals that borrowing constraints alone are able to alter the predictions of the PIH by making the marginal propensity to consume negatively dependent on the change in permanent income. Consequently, even with temporarily high TFP growth, saving still responds positively to income growth. This is illustrated in Figure 5, where the line with squares shows the impulse responses of saving in the partial-equilibrium model to a transitory 1% TFP growth shock, and the line with triangles represents the counterpart general-equilibrium model with borrowing constraints (studied previously in Figure 3) with the deposit rate determined by the marginal product of capital. In this partial-equilibrium experiment the real deposit rate is fixed at 1% and the other parameter values are the same as before (e.g., those generating Figure 3). It is

\[\text{28 When the deposit rate is fixed, whether there is capital or not affects only the income level but not the saving rate.}\]
clear from Figure 5 that saving still responds positively to growth, albeit with smaller magnitudes, even though the change in growth is purely temporary and there is no capital.

![Saving Rate](image)

**Figure 5. Impulse Responses of Saving to 1% Growth Shock**

### 6 The Experience of Japan and China

This section shows that the predictions of the theoretical model are consistent with the high-growth, high-saving, and low-interest rate experience of Japan in the postwar period (1957-2000) and China in the past 30 years. Since the data from Japan are more reliable and less controversial, we discuss the Japanese data in more detail.

**The Case for Japan.** Aoki (1986, p. 579) notes that Japanese households hold most of their savings in the form of safe assets such as bank deposits and postal savings accounts despite the much higher after-tax returns on stockholdings. According to the survey by the Central Council for Financial Services Information in Japan, even as recently as in 2004, the share of deposits in total household financial assets was 41.5%, while the share of postal savings was 18.6%. Therefore, the share of bank deposits and postal savings together accounted for 60.1% of household financial assets (Kishi, 2005, p.808). This ratio was more than 65% in 1970. Kishi (2005, p. 809) also notes that "In regard to the Japanese household portfolio, safety is the highest priority." Horioka (1990) notes that interest rates on bank and postal deposits have been regulated at relatively low levels,
but the rates of return on corporate equities have been very high. One would therefore expect the much higher rates of return on equities to lead to a higher share of the household’s portfolio being held in such assets. However, the share of equities is quite low (Horioka, 1990, p. 83). For example, the share of equities was 7.4% in 2001 (Korb, 2001). Such empirical evidence suggests that deposit rates, instead of the rates of returns to capital, were the most relevant interest rates for household saving in Japan in the postwar period.

Figure 6 plots the 3-month nominal deposit rate (dotted lines), the annual rate of TFP growth (solid lines), the after-tax real rate of returns to capital (dashed lines), and the saving rate (dot-dashed lines) for the period 1957-2000. Most of the data are based on those in Chen et al. (2006).\textsuperscript{29} We use the short-term deposit rate to represent the interest rate that Japanese households face when making saving (deposit) decisions.\textsuperscript{30} The after-tax rate of returns to capital, on the other

\textsuperscript{29}More specifically, the TFP growth and saving rate are exactly identical to those used in Chen et al. (2006), the interest rate is measured as the 3-month deposit rate and is directly from the Bank of Japan. Also plotted in Figure 6 is the after-tax real return to capital. I thank Kaiji Chen for sharing most of the data with me.

\textsuperscript{30}Time series of longer-term deposit rates are not available or have a much shorter sample size. However, the 3-month T-bill rate in Japan behaves similarly to the 3-month deposit rate in the sample period. Patrick and Rosovsky (1976, p.261) shows that the nominal 1-year deposit rate was essentially fixed at 5.5% per year during the 1960-1972 period.
hand, represents the true measure of the marginal products of capital in the economy. Since the inflation rate is extremely volatile in the sample period, we show only the nominal deposit rate in the figure. The average inflation rate in the sample period (above 6% per year) is significantly higher than the deposit rate, so the average 3-month real deposit rate is negative, about −2%.\footnote{Horiuchi (1984) argues that the interest rates in Japan in the postwar period were not particularly low compared with those in the U.S. and developed European countries. However, with respect to Japan’s extraordinary high growth and rates of return to capital, the interest rates in Japan were indeed very low.}

It is clear from the figure that the first three time series (TFP, saving, and capital returns) tend to move together, in both long-term trends and short-term fluctuations. For example, there appears to be a structural change in the long-term trend with a downward shift in the mean of the three time series around the 1973-1974 oil-crisis period. The mean for each of the three series is substantially higher before the crisis (4.5%, 18% and 15%, respectively) than after (1%, 10% and 5%, respectively). Also, in the pre-1973 subsample the twin peaks in the capital returns and saving rate around 1962 and 1969 appear to be driven by the two TFP-growth peaks preceding those years. In contrast, the deposit rate is not only substantially lower than the other three series but does not appear to move closely with any of them over time. In fact, the nominal deposit rate remained roughly constant at 4% before 1973 during the high-growth and high-saving period, and the real deposit rate is negative for that period because of inflation.\footnote{According to Horiuchi (1984), the 6-month deposit rate was also stable with an average of about 5% per year in that period.}

These facts suggest that Japan’s aggregate saving rate (which is essentially the investment-to-output ratio) is fundamentally driven by returns to capital, which in turn is fundamentally driven by TFP growth. The interest rate facing households, on the other hand, does not play an important role in determining the movements in the other series, especially the saving rate. The spread between the real deposit rate and the real rate of return to capital is extremely large, more than 20 percentage points during the high-growth period of the 1960-1970s.

\begin{table}[!h]
\centering
\begin{tabular}{cccccccc}
\hline
\textit{j} & 4 & 3 & 2 & 1 & 0 & −1 & −2 & −3 & −4 \\
\hline
\textit{corr}(g_{t-j}, r^k_t) & 0.48 & 0.60 & 0.63 & 0.70 & 0.69 & 0.39 & 0.29 & 0.31 & 0.16 \\
\hline
\textit{corr}(r^k_{t-j}, \tau_t) & 0.75 & 0.81 & 0.81 & 0.84 & 0.81 & 0.62 & 0.45 & 0.30 & 0.17 \\
\hline
\textit{corr}(g_{t-j}, \tau_t) & 0.49 & 0.53 & 0.54 & 0.58 & 0.47 & 0.09 & 0.00 & 0.03 & -0.06 \\
\hline
\end{tabular}
\caption{Cross Correlations}
\end{table}
and capital returns) are reported in Table 4, where \( g \) stands for TFP growth, \( r^k \) for real rate of returns to capital, and \( s \) for saving rate.\(^{33}\) The subscript \( t - j \) stands for the time period shifted back by \( j \) years. It shows that the contemporaneous correlation between TFP growth and the other two series are strong, 0.69 for capital returns and 0.47 for the saving rate. Also, these correlations are even stronger at \( j = 1 \) (between past TFP growth and the future values of these two series), but much weaker at \( j = -1 \) (between future TFP growth and the past values of these series).

For example, the correlation of \( g_{t-1} \) with \( r^k_t \) is 0.70, and with \( s_t \) is 0.58. On the other hand, the correlation of future TFP growth with the past values of these two series are much weaker, 0.39 for capital returns and 0.09 for saving. The same lead-lag relationship also exists between capital returns and saving. For example, the contemporaneous correlation between the rate of returns to capital and saving is 0.81, but the correlation of past returns to capital with current saving is even stronger, 0.84; and that between future returns to capital and current saving is only 0.62. To sum up, Table 4 indicates that TFP growth may drive (cause) the rate of returns to capital, which in turn may drive the saving rate.

These lead-lag relationships are consistent with the prediction of standard neoclassical growth theory that TFP growth should lead to increased saving by raising the equilibrium real interest rate (the marginal product of capital). However, the marginal product of capital may not be the interest rate faced by Japanese households. The figure shows that the real deposit rate in Japan was negative during the high-growth high-saving period (because the average inflation rate was more than 6% per year). Hence, the real puzzle is why Japanese households were willing to save so much to finance the country’s high investment demand when the returns to their savings were so low. The model presented in the previous sections offers a plausible answer: Borrowing Constraints.

**The Case for China.** According to Kraay (2000), between 1978 and 1995, the main source of the increase in household saving was the rapid growth of household deposits in the banking system, which accounts for the bulk of the increase in the saving rate. For example, by 1995 the net change in deposits was more than three times larger than individual investment. Wei and Zhang (2009) documents that household saving rate rose from 16% in 1980 to 30% in 2007, and they also argue that corporate saving is not yet quantitatively as important as household saving in modern China.

Xie (1992) presents data for the structure of household financial assets in China during the 1978-1991 period. The data show that cash and deposits accounted for 100% of total household financial assets in 1978, and this number remained as high as 90% in 1991. Yi and Wang (2008) present data for the 1991-2007 period. Their data show that bank deposits accounted for 72% of total household financial assets in 2004 and 2005. In contrast, the share of bonds and stocks

\(^{33}\) The correlations between deposit rate and the other three time series are essentially zero and are hence not reported.
accounted only for 3.5% and 6.3%, respectively, in 2004; and 3.1% and 5.5%, respectively, in 2005. On the other hand, bank loans have been the major source of external funds for non-financial firms. For example, the share of bank loans in total corporate debt was 63.3% in 2004 and 64% in 2005. In contrast, the share of corporate bonds and stocks was 0.6% and 15.3%, respectively, in 2004; and was 1.3% and 12.8%, respectively, in 2005.

Hence, bank deposits have been not only the major means of saving for Chinese households, but also the most important source of external funds for Chinese firms. Therefore, deposit rates are the most relevant interest rates for saving and borrowing decisions in China, instead of the rates of returns to capital.\textsuperscript{34}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure7.png}
\end{figure}

Figure 7 presents the nominal 1-year deposit rate (dotted lines), the real GDP growth rate (solid lines), gross investment-to-GDP ratio (dot-dashed lines), and real rate of return to capital (dashed lines). Data on nominal deposit rates are not available until 1991, so they are presented in the right window as the dotted line.\textsuperscript{35,36}

\textsuperscript{34}Wei and Zhang (2009) documents that household saving rate rose from 16% in 1980 to 30% in 2007, and they also argue that corporate saving is not yet quantitatively as important as household saving in modern China.\textsuperscript{35} Aggregate employment data and price index for China are highly inaccurate. Hence, it is difficult to obtain reliable estimates of the TFP series and the inflation rate.\textsuperscript{36} Data source: GDP and aggregate investment are from China Statistical Yearbook 2007. The rate of return to
The figure shows that the movements in the aggregate saving rate (i.e., the investment-output ratio) follow closely with those in the rate of return to capital and GDP growth. This is consistent with the neoclassical story that high growth leads to high saving through high marginal product of capital. Although the interest rate also tends to move with GDP growth, it is substantially lower than the rate of return to capital; the spread is about 20 percentage points and this large level of spread has remained roughly stable over time, although after 1995 both the interest rate and the rate of return to capital have declined visibly. However, since 2001, the rate of return to capital has risen again but the interest rate has remained roughly constant at about 2% per year. The national saving rate, on the other hand, is clearly one of the highest in the world, with an average of 37%, and has shown no sign of diminishing over time. In fact, it has risen above the 40% level since 2003.

Such an extremely high saving rate in conjunction with a low interest rate is difficult to reconcile with standard theories. According to the conventional "saving-cause-growth" theory, high growth is the result of high saving. But this theory cannot explain the Chinese experience where the marginal product of capital remains high and the interest rate remains low. In particular, where does the high saving come from given the low interest rate? On the other hand, according to the "growth-cause-saving" theory, high growth leads to high returns to capital, hence stimulating saving through a high equilibrium interest rate. But the empirical evidence for a positive correlation between saving and interest rates is generally very hard, if not impossible, to find; and worse still, interest rates in China and other fast-growing economies are typically very low.

7 Conclusion

TFP growth promotes investment, which in turn stimulates saving through a general-equilibrium endogenous interest rate effect. However, fast-growing economies tend to have not only high saving rates but also extremely low real interest rates. Hence, the real puzzle is not why high growth can lead to high saving, but why saving can be so high when the interest rate is so low.

This paper uses a growth model with borrowing constraints to show that high growth can still lead to high saving even though the real interest rate facing households is low. The reason is that borrowing constraints can completely alter the relationship between the marginal propensity to consume and permanent income, so that higher permanent income leads to increased saving, instead of higher consumption, in sharp contrast to the predictions of the PIH. Therefore, my model suggests that two factors may have played decisive roles for fast-growing economies’ high saving rates: (i) borrowing constraints and (ii) productivity growth.

capital is from Bai, Hsieh, and Qian (2006). The deposit rate is from People’s Bank of China. Note that the data series for the rate of return to capital ends in 2005. The last 2 years of data (for 2006 and 2007) are based on the projection that the rate shall remain at the 2005 level of 21%. 
Because precautionary saving under borrowing constraints can support a large spread between the deposit rates and the rates of return to capital, one immediate implication is that if the spread in rates of return due to financial repressions and institutional distortions in China were eliminated by allowing interest rates to rise to market levels, the saving rate in China would rise even further, and such a higher saving rate is welfare improving, other things equal. This suggests that the current 40% saving rate in China may not be high enough, but lower than it could be, in sharp contrast to the popular views in the profession and policy circles. That China’s rate of return to capital has been so high (about 23% per year) and shown little sign of diminishing despite the above 40% investment rate is consistent with this hypothesis.37

The model also offers a simple framework for studying wealth distribution and welfare implications. The analytical tractability of the model can greatly simplify the computation of welfare in the presence of growth and borrowing constraints. As a future research topic, for example, it would be interesting to compute the welfare gains of eliminating the spread between the deposit rate and the rate of return to capital in China and how this would affect the wealth distribution.

The model has some limitations because of the simplifying assumptions involved. For example, as in the standard literature (e.g., Carroll et al. 2000; Chen, et al. 2006), the model does not distinguish household saving from national saving. That is, firms’ investment is assumed to be financed entirely by household savings. Although household saving is the single most important component of aggregate saving that differentiates China from the developed countries such as the U.S., corporate saving nonetheless is also important and it has risen rapidly in recent years. Also, in the model the implied elasticity of labor supply is infinite in order to render the model analytically tractable. Although developing countries such as China may have a highly elastic labor supply, this certainly does not last forever and does not apply to developed countries. If this assumption is relaxed, complicated numerical solution methods must be used to solve the model. In addition, we have assumed that the support of the idiosyncratic shocks has a lower bound of zero, $\varepsilon(i) \in [0, \varepsilon_{\text{max}}]$, to make closed-form solution possible. This zero lower bound implies that households may face zero consumption because of zero wealth-income, although the probability of this event is zero. However, this is inessential for our results. In the Appendix of this paper, it is shown that when the utility function is more general so that zero consumption is tolerable (i.e., if the coefficient of risk aversion $\gamma < 1$), the results continue to hold. Nonetheless, this paper is not able to fully account for the 40% saving rate observed in China. Without the spread in the rates of return, the model with borrowing constraints implies that, under a 10% annual growth rate, the national saving rate is about 30%; whereas with low and fixed deposit rates, the implied saving rate is between

37 On the other hand, financial development is also expected to reduce household income uncertainty and borrowing constraints, thus ultimately bringing down China’s high-saving and high-capital returns to levels justifiable by TFP growth.
20% to 30%. There are many other factors besides TFP growth and precautionary saving motives that can affect aggregate saving, such as corporate saving, international trade, and public finance. These factors are not analyzed in this paper. In addition, as a first-order approximation and a first step in analyzing the effects of borrowing constraints on the growth-to-saving relationship in an infinite-horizon model, this paper has considered only situations around the balanced growth path. These issues can be further addressed in future studies by extending the current simple framework to a richer environment.

Appendix

This appendix shows that the results obtained in this paper do not hinge on the log utility function. Namely, the results are not dictated by the special feature that the coefficient of relative risk aversion equals 1.

Suppose the utility function is given by $U(C, N) = \frac{C^{1-\gamma}}{1-\gamma} - aN$, where $\gamma \in (0, \infty)$ measures the degree of risk aversion. Since consumption grows over time, if the parameter $a$ is constant, the relative weight of leisure will shrink to zero, unless $a$ also grows over time accordingly. Hence, to ensure balanced growth, assume $a = (1 + \bar{g})^{t(1-\gamma)}$. This implies that leisure time as a fraction of time endowment is constant over time, consistent with the empirical evidence (see, e.g., Ramey and Francis, 2008). By the same transformation as in the main text, we have $U(C, N) = (1 + \bar{g})^{t(1-\gamma)} \left[ \frac{C^{1-\gamma}}{1-\gamma} - N \right]$. Thus, household $i$'s optimization problem becomes:

$$\max_{(c,s)} E_0 \left\{ \max_{\{N\}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t (1 + \bar{g})^{t(1-\gamma)} \left[ \frac{c_t(i)^{1-\gamma} - 1}{1-\gamma} - N_t(i) \right] \right\} \right\}$$

subject to

$$c_t(i) + (1 + \bar{g})s_{t+1}(i) \leq \varepsilon_t(i) [(1 + r_t)s_t(i) + w_tN_t(i)]$$
$$s_{t+1}(i) \geq 0.$$  \hspace{1cm} (53) 

For the households, except the utility function, the economic environment is exactly the same as in the previous model.\(^{38}\) Therefore, the decision rules of an individual’s consumption and saving plans are also characterized by a cutoff strategy and are nearly identical to those in the benchmark model. Hence, the derivation steps are omitted.

The system of equations that determine the general equilibrium of the model is summarized by:

$$\frac{1}{w_t} = \left[ \beta E_t \frac{1 + r_{t+1}}{(1 + \bar{g})^{t+1} w_{t+1}} \right] R(\varepsilon_t^i)$$

\(^{38}\)Without loss of generality, we set $\theta = 0$. 

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\begin{equation}
(1 + r_t) k_t + w_t N_t = \left[w_t R(\varepsilon^*)\right]^\frac{1}{\gamma} \frac{1}{\varepsilon^*_t} \tag{56}
\end{equation}

\begin{equation}
c_t = D(\varepsilon^*_t) [(1 + r_t) k_t + w_t N_t] \tag{57}
\end{equation}

\begin{equation}
(1 + \bar{g}) k_{t+1} = H(\varepsilon^*_t) [(1 + r_t) k_t + w_t N_t] \tag{58}
\end{equation}

\begin{equation}
w_t = (1 - \alpha) \frac{y_t}{N_t} \tag{59}
\end{equation}

\begin{equation}
r_t + \delta = \alpha \frac{y_t}{k_t} \tag{60}
\end{equation}

\begin{equation}
c_t + (1 + \bar{g}) k_{t+1} - (1 - \delta) k_t = y_t \tag{61}
\end{equation}

\begin{equation}
y_t = k_t^\alpha (z_t N_t)^{1-\alpha} \tag{62}
\end{equation}

where

\begin{equation}
R(\varepsilon^*_t) \equiv \left[ \int_{\varepsilon < \varepsilon^*} \varepsilon^{\gamma} \varepsilon(i)^{1-\gamma} dF(\varepsilon) + \int_{\varepsilon \geq \varepsilon^*} \varepsilon(i) dF(\varepsilon) \right], \tag{63}
\end{equation}

and the functions \{D(\varepsilon^*), H(\varepsilon^*)\} are the same as in equations (21) and (22). Note the function \(R(\varepsilon^*)\) differs from the previous model unless \(\gamma = 1\). In particular, the value of \(R\) exceeds that in the previous model if \(\gamma > 1\), and it also increases with \(\gamma\).

In the steady state, we have

\begin{equation}
(1 + \bar{g})^\gamma = \beta (1 + r) R(\varepsilon^*) \tag{64}
\end{equation}

\begin{equation}
c = D(\varepsilon^*) x \tag{65}
\end{equation}

\begin{equation}
(1 + \bar{g}) k = H(\varepsilon^*) x \tag{66}
\end{equation}

\begin{equation}
c + \bar{g} k = y - \delta k, \tag{67}
\end{equation}

where \(x = (1 - \delta) k + y\) is the wealth-income. Define the disposable income \(\varphi \equiv y - \delta k\). Thus, the consumption and saving functions are the same as before:

\begin{equation}
c = \frac{(1 + \bar{g}) D}{\bar{g} + D} \varphi \tag{68}
\end{equation}

\begin{equation}
\bar{g} k = \left[ 1 - \frac{(1 + \bar{g}) D}{\bar{g} + D} \right] \varphi. \tag{69}
\end{equation}

Therefore, the national saving rate is given by the same function as in the previous model:

\begin{equation}
\tau = 1 - \frac{(1 + \bar{g}) D(\varepsilon^*)}{\bar{g} + D(\varepsilon^*)}. \tag{70}
\end{equation}
However, because the function $R$ in equation (63) depends on the value of $\gamma$, the cutoff $\varepsilon^*$ will also be a function of $\gamma$, as is the saving rate. The equation that determines the value of the cutoff is given by

$$\frac{(1 + \tilde{g})^\gamma}{R(\varepsilon^*)} = \beta \left[ 1 - \delta + \alpha \left( \tilde{g} + \delta + (1 + \tilde{g}) \frac{D(\varepsilon^*)}{H(\varepsilon^*)} \right) \right],$$

which is analogous to equation (35). Assuming that $\varepsilon$ follows the same distribution as in the previous model, we have

$$R(\varepsilon^*) = 1 + \left[ \frac{\sigma^\gamma}{(1 - \gamma + \sigma)(1 + \sigma)} \right] \varepsilon^{\sigma \max \varepsilon^{1+\sigma}}$$

$$D(\varepsilon^*) = \varepsilon^* \left[ 1 - \frac{1}{1 + \sigma} \varepsilon^{\sigma \varepsilon^{\sigma}} \right]$$

$$H(\varepsilon^*) = 1 - \varepsilon^* \left[ 1 - \frac{1}{1 + \sigma} \varepsilon^{\sigma \varepsilon^{\sigma}} \right].$$

Notice that we must require $\gamma < 1 + \sigma$ to ensure that the function $R(\varepsilon^*)$ is greater than one. This is good news because the value of $\sigma \in (0, \infty)$ and the inequality of the wealth distribution in the current model depends only on the relative magnitude $|\sigma - \gamma|$. Hence, to generate the same Gini coefficient in wealth distribution when the value of $\gamma$ is larger than one, we can simply raise the value of $\sigma$ accordingly. In other words, with a large degree of risk aversion, even a small variance in the idiosyncratic wealth shocks can generate a large enough inequality across households.\textsuperscript{39}

Nonetheless, the results of the paper are stronger the smaller the value of $\gamma$. This implies that assumption of a zero lower bound on the support of the distribution, $\varepsilon(i) \in [0, \varepsilon_{\max}]$, is inessential, even though this may imply close-to-zero consumption if the realization if $\varepsilon$ is low (note the probability of $\varepsilon(i) = 0$ is zero). Close-to-zero consumption implies extremely negative utilities when $\gamma \geq 1$. However, if $\gamma < 1$, then close-to-zero consumption can be tolerated by the utility function. Hence, the problem occurs only when $\gamma \geq 1$. Since a larger value of $\gamma$ will weaken our results (as the following example shows), this zero-lower-bound assumption is therefore not what is driving our results.

Let the structural parameters take the following values: $\beta = 0.96$, $\delta = 0.1$, and $\alpha = 0.4$, as in the previous model. Also let the coefficient of risk aversion be $\gamma = 5$, which is a sufficiently large number. With this value of risk aversion, we set $\sigma = 4.05$, which satisfies the requirement $\gamma < 1 + \sigma$ and implies a sufficiently large Gini coefficient. Figure 8 (the left-hand window) shows the relationship between saving and growth, where the line with circles represents the standard growth

\textsuperscript{39}Recall that the variance $\varepsilon$ in the power distribution is inversely related to $\sigma$. The variance approaches zero when $\sigma \to \infty$.  

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without borrowing constraints and the line with triangles the counterpart model with borrowing constraints. In both models $\gamma = 5$.

The left-hand window shows that (i) high risk aversion reduces saving at all levels of growth, other things equal (compared with Figure 1); (ii) saving and growth are still positively related, although the positive relation is much weaker, regardless of borrowing constraints; and (iii) with borrowing constraints, this positive relation is significantly magnified. The stronger the growth, the larger the amplification. For example, when the growth rate increases from 1% to 10% a year, the saving rate rises from 2.5% to 5.4% in the standard model without borrowing constraints, but under borrowing constraints the saving rate rises from 3% to 12%.

The right-hand window in Figure 8 shows that the marginal product of capital rises too fast without borrowing constraints, while this is not the case with borrowing constraints. Hence, borrowing constraints greatly mitigate the growth effect on the real interest rate.

![Figure 8. The Growth Effects on Saving and Interest ($\gamma = 5.0$)](image)

To see why the relationship between saving and growth remains positive even under high de-
degrees of risk aversion, consider the saving rate in the standard growth model without borrowing constraints:

\[ \tau = \frac{\bar{g}k}{\varphi} = \frac{\bar{g}\beta\alpha}{(1 + \bar{g})^\gamma - \beta(1 - \delta) - \delta\beta\alpha}. \]

Differentiating this expression with respect to \( \bar{g} \) yields

\[ \frac{d\tau}{d\bar{g}} = \beta\alpha \frac{[(1 + \bar{g})^\gamma - \beta(1 - \delta) - \delta\beta\alpha - \gamma(1 + \bar{g})^\gamma - 1]}{[(1 + \bar{g})^\gamma - \beta(1 - \delta) - \delta\beta\alpha]^2}. \]

The sign of \( \frac{d\tau}{d\bar{g}} \) depends only on the numerator of this expression, which is positive if

\[ (1 + \bar{g})^{-1} [1 + \bar{g} - \bar{g}\gamma] > \beta(1 - \delta) + \delta\beta\alpha. \]

The left-hand side of this inequality decreases with \( \bar{g} \) and \( \gamma \). Suppose \( \gamma = 5 \), \( \bar{g} = \delta = 0.1 \), \( \alpha = 0.4 \), and \( \beta = 0.96 \), then the left-hand side of the above expression takes the value 0.9317, whereas the right-hand side takes the value 0.9024. Hence, for values of \( \gamma \) within the empirical range (i.e., \( \gamma \in (0, 5) \)), the relationship between saving and growth is positive even in a standard growth model, albeit a weak one if \( \gamma = 5 \). This positive relation can be greatly amplified by borrowing constraints.
References


